Production Risk and Optimal Input Decisions

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The paper examines the impact of production risk on a producer's optimal input decisions. Whether producers use more or fewer inputs in a yield-risky environment depends on the sign of the marginal risk premium, which is determined by risk preferences and technology. I present the weakest condition on technology that is sufficient to sign the marginal risk premium for all risk-averse preferences. If this condition fails to hold, the marginal risk premium is not of the same sign for all risk averters. Results are used to explore the properties of an estimated technology.

Key words: marginal risk premium, production risk, risk aversion.

The principal question that this paper addresses is how does production uncertainty affect optimal input decisions? As is well known, risk-averse decision rules differ from risk neutral choices because of the existence of a marginal risk premium, which is the wedge between input cost and expected marginal product at the optimum level of input use. The sign of the marginal risk premium indicates whether risk-averse producers use more or fewer inputs than risk-neutral producers. In general, the sign depends on risk preferences and technology. Because the latter is observable and often measurable, it is useful to work out the implications for optimal input choices using properties of the technology, while relying as little as possible on information about preferences. The present paper discovers the weakest condition on the technology sufficient to sign the marginal risk premium for all risk-averse preferences.

Previous results (MacMinn and Holtmann, Pope and Kramer) have been obtained using production function representations. Output is a function of a single input and a random shock. In such a setup, the marginal risk premium is of the same sign as the covariance between marginal utility and marginal product. Because marginal utility is decreasing for risk-averse preferences, a sufficient condition for signing the marginal risk premium is monotonicity of the marginal product in the output shock. The sufficient condition on technology discovered here is weaker than such a monotonicity restriction and therefore applicable to a greater range of technologies. Moreover, it cannot be weakened any further as it is also a necessary condition for the marginal risk premium to be of the same sign for all concave utility functions.

I also provide an economic interpretation of the above necessary and sufficient condition. The condition, it turns out, is equivalent to requiring that an input be either risk-increasing or risk-decreasing. My definition of risk-increasing and risk-decreasing inputs is derived from the Rothschild and Stiglitz definition of increasing risk and is therefore different from that of Pope and Kramer.

My principal result thus proves that, for all risk averters, marginal risk premium is positive (negative) if and only if the input is risk-increasing (decreasing). It follows that, if the input is neither risk-increasing nor risk-decreasing, the marginal risk premium is not of the same sign for all risk averters. For such situations, the paper derives a sufficient condition on technology which signs the marginal risk premium for the restricted class of concave utility functions with convex marginal utility.

Finally, I illustrate the use of the theoretical conditions in an empirical example. This is done with the help of the conditional distribution functions which Taylor estimated for corn and cotton on the basis of an experimental fertilizer response data set first used by Day. In this example, the monotonicity restrictions completely fail to sign the marginal risk premium, and it is

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This paper has its origins in chapter 4 of the author's dissertation at the University of Minnesota. For encouragement and valuable criticism, the author is grateful to Theodore Graham-Tomasi, Terry Roe, Jan Werner, and two anonymous reviewers.

Review coordinated by Steven Buccola.

necessary to make use of the weaker condition discovered here. Even then, there are instances where the input fails to be risk-increasing or risk-decreasing. In such cases, the properties of the estimated technology are further examined for the purpose of signing the marginal risk premium for risk averters with convex marginal utility.

All proofs are collected together in the appendix.

Risk Aversion and Optimal Input Choice

Output q is assumed to be a random variable with a conditional cumulative density function $F(q, \mathbf{x})$, where $\mathbf{x}$ is a vector of inputs.[1] Let x_i be the ith element of the input vector $\mathbf{x}$. Here, F is assumed twice differentiable in x_i and q and, in particular, the partial derivatives F_q, $F_{\mathbf{x}_i}$, F_{qq}, and $F_{\mathbf{x}_i q}$ exist. Also, it is assumed that for all input applications the support of the distribution function F is contained in a compact interval $[q_0, q_m]$; that is, $F(q_0, \mathbf{x}) = 0$ and $F(q_m, \mathbf{x}) = 1$ for all input vectors $\mathbf{x}$. It follows that the gradient vectors $F_x(q_0, \cdot)$ and $F_x(q_m, \cdot)$ are each row vectors with zeros everywhere. For output levels between the two end points of q_0 and q_m, I assume $F_{\mathbf{x}_i}(q, \mathbf{x}) < 0$, that is, increasing input use leads to a superior output distribution in the sense of first degree stochastic dominance.[2]

If $\mathbf{w}$ is a vector of input prices and output price is normalized at unity, applying inputs $\mathbf{x}$ yields profit $\pi(q, \mathbf{x}) = q - \mathbf{w}'\mathbf{x}$. The optimal input vector is found by maximizing the expected utility of profit, where utility is assumed increasing and concave.

Let RP be the risk premium that a risk-averse producer is willing to pay for removing all uncertainty in output. Because the output distribution is conditional on input use, RP is a function of input vector $\mathbf{x}$. Further, RP satisfies $\mathrm{E}U[\pi(q, \mathbf{x})] = U\,[\mathrm{E}\pi(q, \mathbf{x}) - RP(\mathbf{x})]$. Maximizing expected utility is equivalent to maximizing the certainty equivalent; namely, expected profit net of the risk premium. Letting $\mathbf{x}^*$ be the optimal input application, the first-order condition to the latter problem is

$$(\mathrm{E}\pi)_{\mathbf{x}_i} - RP_{\mathbf{x}_i}(\mathbf{x}^*) = 0$$

$$\text{or } -w_i + \int (q - w'\mathbf{x}^*)F_{qx_i}(q, \mathbf{x}^*)dq = RP_{\mathbf{x}_i}(\mathbf{x}^*)$$

$$\text{or } \int \pi(\mathbf{x}^*)F_{qx_i}(q, \mathbf{x}^*)dq = w_i + RP_{\mathbf{x}_i}(\mathbf{x}^*).$$

Here, RP_{x_i}, known as the marginal risk premium for input i (Pope and Kramer, MacMinn and Holtmann) is the wedge between input cost and expected marginal product at the optimum level of input use. If the producer is risk-neutral, the marginal risk premium is zero. For risk-averse preferences, the marginal risk premium is nonzero and its sign is given by proposition 1 below.

The following notation is useful for stating the result. Let $t(q, \mathbf{x}) = F_{x_i}(q, \mathbf{x})/F_q(q, \mathbf{x}) - \mathrm{E}[F_{\mathbf{x}_i}(q, \mathbf{x})/F_q(q, \mathbf{x})]$, and $T(q, \mathbf{x}) = \int_{q0}^{q} t(y, \mathbf{x})F_q dy$.

PROPOSITION 1: *For all concave utility functions $RP_{\mathbf{x}_i}(x^*)$ is strictly positive (negative) if and only if*

(1) $T(q, \mathbf{x}^*) \geq (\leq)\ 0$ *for all* $q \in (q_0, q_m)$.

Inequality (1) is a necessary and sufficient condition for the marginal risk premium to be of uniform sign for all risk-averse utility functions. Furthermore, because (1) represents a condition on the distribution function, it is the weakest condition on technology that signs the marginal risk premium for all risk-averse preferences.[3] The next section considers an economic interpretation of (1) in terms of the impact of input use on output variability.

Inequality (1) may be implied by stronger restrictions on technology. It is useful to state one such restriction because of its value in empirical illustration and in allowing comparisons of (1) with earlier work.

PROPOSITION 2: *$T(q, \mathbf{x}) \geq (\leq)\ 0$ for all q, if $t(q, \mathbf{x})$ has one root over its output domain (say, q_I) such that*

$$t(q, \mathbf{x}) = F_{x_i}(q, \mathbf{x})/F_q(q, \mathbf{x}) - \mathrm{E}(F_{x_i}(q, \mathbf{x})/F_q(q, \mathbf{x})) > (<)\ 0 \text{ for } q \in (q_0, q_I)$$

and

$$t(q, \mathbf{x}) = F_{x_i}(q, \mathbf{x})/F_q(q, \mathbf{x}) - \mathrm{E}(F_{x_i}(q, \mathbf{x})/F_q(q, \mathbf{x})) < (>)\ 0 \text{ for } q \in (q_I, q_m).$$

Notice that the above condition is always sat-

[1] For convenience, notation $F(q, \mathbf{x})$ has been used instead of $F(q|\mathbf{x})$ to denote the conditional distribution function. Because $\mathbf{x}$ is not a random variable, $F(q, \mathbf{x})$ should not be interpreted as a joint distribution.

[2] If the conditional distribution function F is induced by an underlying production function of the form $z(\mathbf{x}, \theta)$, where θ is the random shock, then $F_x < 0$ is equivalent to the assumption of positive marginal productivity; that is, $z_x(\cdot, \theta) > 0$ for all θ.

[3] The exception is when optimal risk-averse input choice $x_i^* = 0$. Then the marginal risk premium is negative when $T(q, \mathbf{x}) \leq 0$ for all q, but it could be of either sign if $T(q, \mathbf{x}) \geq 0$ for all q.

isfied if $F_{x_i}(q, \mathbf{x})/F_q(q, \mathbf{x})$ is monotonic decreasing (increasing) in q.

Relationship to Earlier Findings

The sufficiency part of proposition 1 generalizes earlier results. MacMinn and Holtmann, and Pope and Kramer have examined conditions sufficient to sign the marginal risk premium for risk-averse preferences. Both these papers use a production function representation of the stochastic technology. Let $q = z(\mathbf{x}, \theta)$, where z is the production function and θ the random output shock. The principal result is the following. If $z_{x_{i\theta}}$ is positive (negative) for all θ and for some range of input values containing the optimum, the marginal risk premium is positive (negative) for input i. In this way, knowledge of production function characteristics can be used to predict departures from risk-neutral input use. The result is, however, a special case of proposition 1, and the marginal risk premium can be signed in more general circumstances.

It can be shown[4] that if the conditional distribution function $F(q, \mathbf{x})$ is induced by a stochastic production function $z(\mathbf{x}, \theta)$ with $z_\theta > 0$, then $F_{x_i}/F_q - \mathrm{E}^q[F_{x_i}/F_q]$ is greater than, equal to, or less than zero as $z_{x_i} - \mathrm{E}^\theta[z_{x_i}]$ is less than, equal to, or greater than zero, where the superscript on E indicates the variable over which expectations are taken. A stronger version of this result, which is also true, is that monotonicity of z_{x_i} in θ implies monotonicity of F_{x_i}/F_q in q but in the opposite direction. Now consider hypothetical figures 1 and 2 below. In figure 1, z_{x_i} is an increasing function of θ. This means F_{x_i}/F_q is a decreasing function of q. But by proposition 2, $T(q, \mathbf{x}) \geq 0$ for all q, and hence the marginal risk premium is positive. This is the result of MacMinn and Holtmann, and Pope and Kramer. In figure 2, on the other hand, it is not possible to sign the marginal risk premium using a monotonicity condition on z_{x_i}. But because $z_{x_i} - \mathrm{E}^\theta[z_{x_i}]$ has one sign change from negative to positive, $F_{x_i}/F_q - \mathrm{E}^q[F_{x_i}/F_q]$ has also one sign change from positive to negative. This means, by proposition 2, that $T(q, \mathbf{x}) \geq 0$ for all q, implying the marginal risk premium is positive. Proposition 2 can therefore sign the marginal risk premium in more instances than do the monotonicity conditions of earlier work. Of course, the sufficient condition of proposition 2 is in turn a stronger restriction than the condition in proposition 1.

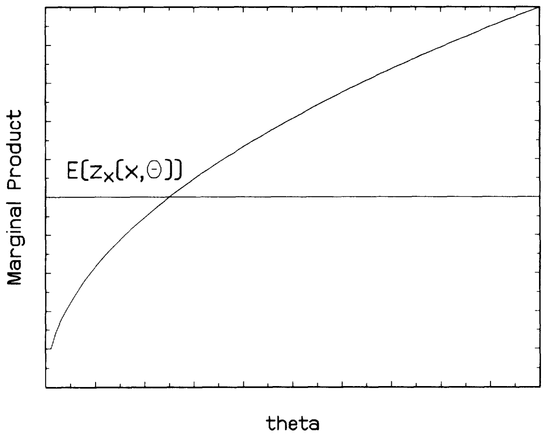

Figure 1. Monotone marginal product

Inequality (1) cannot be weakened further according to the necessity part of proposition 1. This result has so far not been known or proved. The implication is that if $T(q, \mathbf{x}^*)$ fails to be of uniform sign over its domain, the marginal risk premium is also not of uniform sign for all risk-averse utility functions. In other words, the sign of the marginal risk premium may change with the choice of the utility function even within the class of concave functions. This is a disturbing result for empirical work, which often assumes a specific utility functional form. One may therefore wish to discover subsets of the class of concave utility functions for which the marginal risk premium is of uniform sign, even when condition (1) fails. The problem is considered below for an empirically important subclass of concave utility functions.

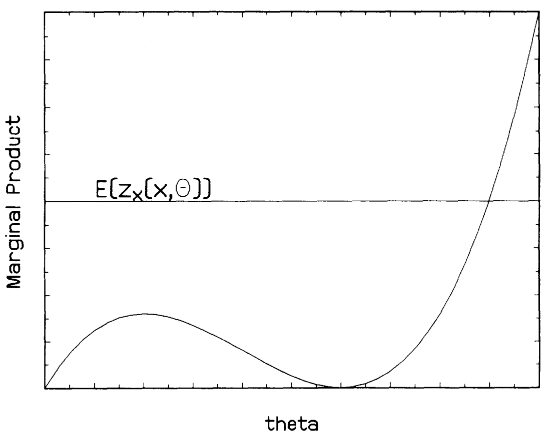

Figure 2. Nonmonotone marginal product

[4] A proof is available from the author on request.

Risk-Increasing and Risk-Decreasing Inputs: An Interpretation

This section advances a definition of risk-increasing and risk-decreasing inputs which, although different from the definition proposed by Pope and Kramer, is consistent with the notion of increasing risk discussed by Rotchschild and Stiglitz. It will then be seen that (1) is precisely the condition for an input to be risk increasing or risk decreasing.

Following Rothschild and Stiglitz, the paper considers one output distribution to be more variable (riskier) than another if they have the same mean and if the riskier distribution has more weight in its tails. An increase in input use, however, increases mean output. To compare distributions with the same mean, consider the distribution of output deviation from its mean. If $\bar{q}(\mathbf{x})$ is mean output, let $h = q - \bar{q}(\mathbf{x})$. The output mean deviation h has zero mean at all input levels. However, because h is a linear transformation of q, its distribution is identical to that of q in all other respects. If $\hat{F}(h, \mathbf{x})$ is the cumulative density of h, then $F(q, \mathbf{x}) = \hat{F}(h, \mathbf{x})$ for all q and $\mathbf{x}$. An input is defined to be risk increasing (or risk decreasing) if it increases (or decreases) the variability of the deviations from output mean. More formally, we have the following definition[5]:

DEFINITION: The ith input is risk-increasing (decreasing) at $\mathbf{x}_0$ if

$$(2) \qquad \int_{q_0}^{q_m} \hat{F}_{x_i}[h(q, \mathbf{x}_0), \mathbf{x}_0]dq = 0$$

and

$$(3) \qquad \int_{q0}^{q} \hat{F}_{x_i}[h(y, \mathbf{x}_0), \mathbf{x}_0]dy \geq (\leq)\ 0 \text{ for all } q \in (q_0, q_m)$$

where $h(q, \mathbf{x}) = q - \bar{q}(\mathbf{x})$ and $\bar{q}(\mathbf{x}) = \int_{q0}^{q_m} qF_q(q, \mathbf{x})dq$.

The condition in (2), which requires that the expected value of h be invariant to changes in input level, is satisfied by construction. The definition essentially amounts to the second condition in (3), which states that an increase in input use always leads to a distribution of deviations from output mean with more (or less) weight in its tails. We now show that (1) is merely another form of (3).

Because $F(q, \mathbf{x}) = \hat{F}(h, \mathbf{x})$ for all q and $\mathbf{x}$, $F_q(q, \mathbf{x}) = \hat{F}_h(h, \mathbf{x})h_q(q, \mathbf{x}) = \hat{F}_h(h, \mathbf{x})$ and $F_{x_i}(q, \mathbf{x}) = \hat{F}_h(h, \mathbf{x})h_{x_i}(q, \mathbf{x}) + \hat{F}_{x_i}(h, \mathbf{x})$. This implies

$$(4) \qquad \hat{F}_{x_i}(h, \mathbf{x}) = F_{x_i}(q, \mathbf{x}) - \hat{F}_h(h, \mathbf{x})h_{x_i}(q, \mathbf{x}) = F_{x_i}(q, \mathbf{x}) + F_q(q,\mathbf{x})\bar{q}_{x_i}(\mathbf{x}).$$

But mean output is $\bar{q} = \int_{q0}^{q_m} qF_q(q,\mathbf{x})dq$. Integrating by parts, $\bar{q} = q_m - \int_{q0}^{q_m} F(q, \mathbf{x})dq$. Hence $\bar{q}_{x_i} = -\int_{q0}^{q_m} F_{x_i}(q, \mathbf{x})dq = -\int_{q0}^{q_m}[F_{x_i}(q, \mathbf{x})/F_q(q, \mathbf{x})]F_q(q, \mathbf{x})dq = -\mathrm{E}[F_{x_i}/F_q]$.

Substituting for $\bar{q}_{x_i}$ in (4) and substituting (4) into (3), an input is risk-increasing at $\mathbf{x}_0$ if

$$(5) \qquad \int_{q0}^{q} \{F_{x_i}(y, \mathbf{x}_0) - F_q(y,\mathbf{x}_0)\mathrm{E}[F_{x_i}(q, \mathbf{x}_0)/F_q(q, \mathbf{x}_0)]\}dy \geq (\leq)\ 0 \text{ for all } q \in (q_0, q_m)$$

Using the notation of $t(q, \mathbf{x})$ and $T(q, \mathbf{x})$ introduced earlier, (5) becomes

$$\int_{q0}^{q} t(y, \mathbf{x}_0)F_q dy \geq (\leq)\ 0 \text{ for all } q \in (q_0, q_m) \text{ or } T(q, \mathbf{x}_0) \geq (\leq)\ 0 \text{ for all } q,$$

which is, of course, the condition in (1). Hence, a restatement of proposition 1 is

PROPOSITION 1*: *For all concave utility functions, the marginal risk premium for input i is strictly positive (or negative) if and only if input i is risk increasing (or risk decreasing) at the optimum level of input use.*

Thus, the definition of risk-increasing and risk-decreasing inputs proposed here provides an economic interpretation of proposition 1. The result is useful because it relates a purely technological characteristic of inputs (as to how they affect output variability) to their use according to risk preferences. But proposition 1* also proves the converse, namely that an input is risk increasing (decreasing) if the marginal risk premium is positive (negative). In Pope and Kramer, on the other hand, an input is defined to be risk increasing (decreasing) if the marginal risk premium is positive (negative). Proposition 1* can therefore be used as the basis for a Pope and Kramer definition if a primitive definition of risk-affecting inputs is in terms of the condition in (1).

Finally, the condition in (1) admits of one more interpretation. If (1) is satisfied, it means that the distributions of deviations from output mean can be ranked in the sense of second degree sto-

[5] The Rothschild-Stiglitz definition of increasing risk compares two distributions. In this paper, however, x parameterizes a family of distributions $F(q, \mathbf{x})$ and the definition is modified accordingly (see Diamond and Stiglitz).

chastic dominance. Increases in risk-increasing inputs lead to less preferred distributions while increases in risk-decreasing inputs lead to more preferred distributions.

Further Assumptions on Preferences or Technology

As noted earlier, if $T(q, \mathbf{x})$ changes sign over its output domain, that is, if the input is neither risk-increasing nor risk-decreasing, the sign of the marginal risk premium depends on the utility function. However, the marginal risk premium may still be of the same sign for a subset of the class of concave utility functions. An empirically important class of functions are those with convex marginal utility because they include all decreasing and constant risk-averse utility functions.

PROPOSITION 3: *The marginal risk premium of input i is strictly positive (negative) for all risk-averse agents with convex marginal utility if*

(6)
$$\int_{q0}^{q} T(y, \mathbf{x}^*)dy \geq (\leq)\ 0 \text{ for all } q \text{ in } (q_0, q_m).$$

Condition (6) is implied by (1) but the converse is not true. Thus, by restricting the set of utility functions, a weaker condition on technology is obtained. If preferences are further restricted to satisfy linear marginal utility (namely, quadratic utility functions), the sufficient condition is even weaker than (6). In this case, the marginal risk premium is strictly positive (negative) if $\int_{q0}^{q_m} T(q, \mathbf{x}^*)dq > (<)\ 0$.

An economic interpretation of (6) is that an increase in input use leads to a less preferred (or more preferred) output distribution according to third degree stochastic dominance. Seen this way, the relationship of (6) to (1) is obvious.

Technology

A functional form popular in the econometric estimation of production functions is to let $q(\mathbf{x}, \theta) = \mu(\mathbf{x}) + \sigma(\mathbf{x})\theta$, where θ is normally distributed with mean zero and unit variance (Just and Pope, Buccola and McCarl). This implies that output q is normally distributed with a mean μ and variance σ^2, where both are conditional on $\mathbf{x}$. In this case, verification of (1) is particularly simple.

Because the area under the normal curve is equal to the corresponding area under the standard normal curve,

$$F(q, \mathbf{x}) = F(z, \mathbf{x}) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-k^2/2)dk, \text{ where } z = \frac{q-\mu(\mathbf{x})}{\sigma(\mathbf{x})}$$

and $F_{x_i}/F_q = [(1/\sqrt{2\pi})\exp(-z^2/2)z_{x_i})]/[(1/\sqrt{2\pi})\exp(-z^2/2)] = z_{x_i}$

$$\text{Now } z_{x_i} = -[\sigma(\mathbf{x})\mu(\mathbf{x})]/\sigma^2 - \sigma_{x_i}[q - \mu(\mathbf{x})]/\sigma^2 = -(\mu_{x_i} + \sigma_{x_i}z)/\sigma$$

$$\text{Hence } t(q, \mathbf{x}) = F_{x_i}/F_q - \mathrm{E}(F_{x_i}/F_q) = -(\mu_{x_i} + \sigma_{x_i}z)/\sigma + \mu_{x_i}/\sigma = -\sigma_{x_i}z/\sigma$$

$$\text{and } T(q, \mathbf{x}) = -(\sigma_{x_i}/\sigma)\int_{-\infty}^{z} k\frac{1}{\sqrt{2\pi}} \exp(-k^2/2)dk.$$

But the quantity inside the integral has a maximum value of zero when the upper limit is plus infinity. Hence T has the same sign as σ_{x_i}. In other words, an input is risk increasing (or risk decreasing) if it increases (or decreases) the variance of output. This result can also be seen directly from the fact that the variance of deviations from mean output is also the variance of output.

An Empirical Illustration

This section presents the results of an empirical investigation into the properties of certain estimated conditional distribution functions. My objective is to sign the marginal risk premium for the estimated technology. The exercise uses Taylor's estimates of conditional distribution functions for corn and cotton. Taylor employed a well known data set of experimental fertilizer response first used by Day to estimate the Pearson system of yield probability density functions. Later, Just and Pope used the same data set to estimate the marginal effects of fertilizer use on output variance. The data consist of corn and cotton yield response to 7 levels of nitrogen application between 0 and 45 lbs per acre at intervals of 7.5. Experiments were carried out for 37 years between 1921 and 1957 at the Delta Branch of the Mississippi Agricultural Experiment Station (Grissom and Spurgeon).

Taylor estimated a conditional cumulative density of the following form: $F(q, x) = 0.5 + 0.5 \tanh[P(x, q)]$, where $P(x, q)$ is a polynomial in x and q and tanh is the hyperbolic tangent given by $\tanh(u) = (e^u - e^{-u})/(e^u + e^{-u})$. The functional forms of P, estimated by Taylor, are

$$P(x, q) = \beta_1 + \beta_2 q + \beta_3 q^2 + \beta_4 q^3 + \beta_5 x + \beta_6 x^3$$

for q = cotton yield and x = nitrogen application rate and

$$P(x, q) = \beta_1 + \beta_2 q + \beta_3 q^2 + \beta_4 q^3 + \beta_5 x + \beta_6 qx + \beta_7 xq^2$$

for q = corn yield and x = nitrogen application rate.

Here, F_x/F_q equals $P_x(x, q)/P_q(x, q)$ and is directly calculated given the parameter estimates, whereas $E(F_x/F_q)$ is computed by numerical integration. The support of q is taken to be the interval between zero and the highest realization of q (for all input applications).[6] Thus, we have $t(q, x) = F_x/F_q - E[F_x/F_q]$ for each of the 7 levels of nitrogen application and for each of the two crops. In no case is F_x/F_q monotonic in q. Sufficient conditions discussed by MacMinn and Holtmann, and Pope and Kramer cannot therefore sign the marginal risk premium.

The next question is whether the technology, despite its nonmonotonicity, satisfies condition (1). We find that in the case of cotton, and for nitrogen applications up to 22.5 lbs per acre, $t(q, x)$ changes sign once from positive to negative, as q increases from zero to its upper support (see figure 3a). But this means T is nonnegative for all q (proposition 2) and hence nitrogen is risk increasing. Beyond 22.5 lbs, however, t changes sign twice (figure 3b). It can be shown that this means the input is neither risk increasing nor risk decreasing; that is, T is not of the same sign over its output domain.[7] The picture is reversed for corn. Here, at low levels of nitrogen the input is neither risk increasing nor risk reducing (figure 4a) while at higher rates (greater than or equal to 30 lbs) it becomes risk increasing (figure 4b). The input is risk increas-

[6] A reviewer has pointed out that the conclusions may be sensitive to this aspect of the empirical procedure.

[7] The relevant result (not proved here because of space constraints) is that if $t(q, x)$ has n finite roots over the interval (q_0, q_m), n is necessarily odd whenever the input is risk-increasing or risk-decreasing.

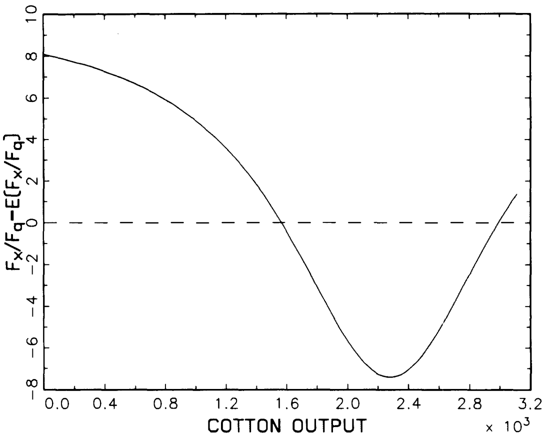

Figure 3b. Effect of nitrogen use (37.5 lbs/acre) on the cumulative distribution of cotton output

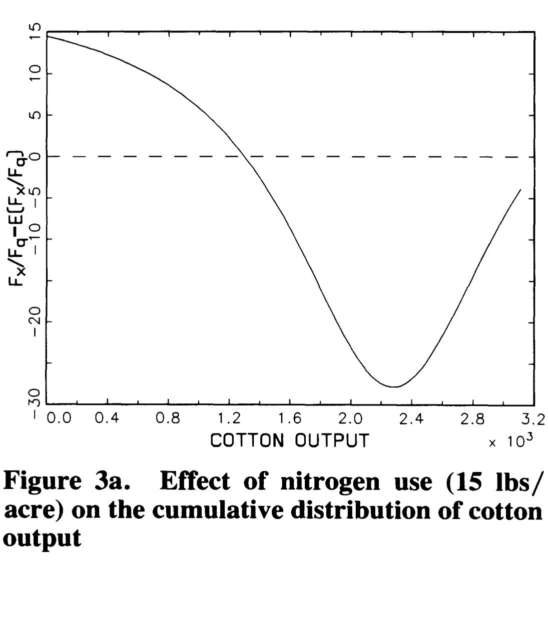

Figure 3a. Effect of nitrogen use (15 lbs/acre) on the cumulative distribution of cotton output

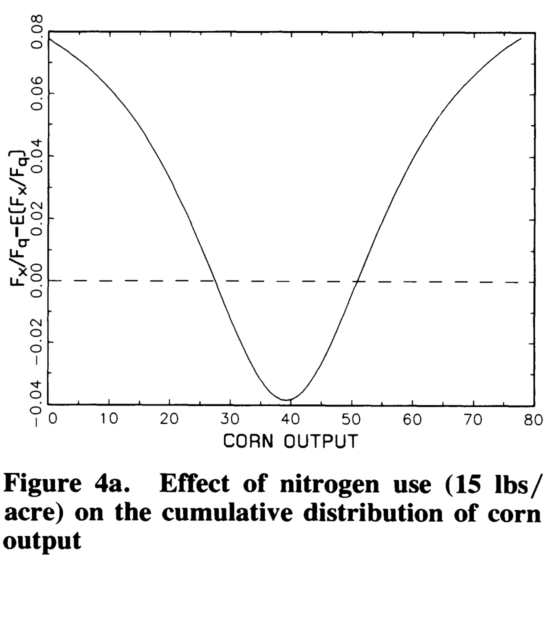

Figure 4a. Effect of nitrogen use (15 lbs/acre) on the cumulative distribution of corn output

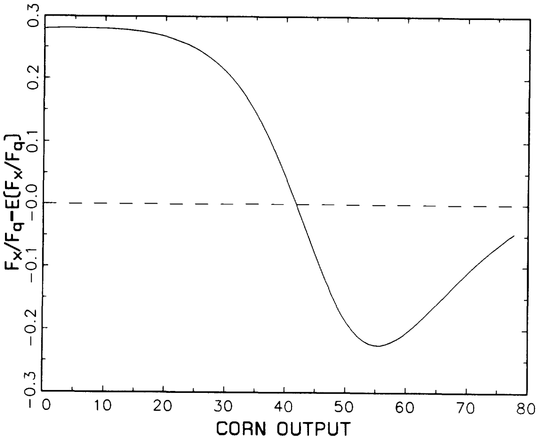

Figure 4b. Effect of nitrogen use (37.5 lbs/acre) on the cumulative distribution of corn output

ing in half of the 14 cases considered and of indeterminate character in the rest of the cases.

In situations where nitrogen is neither risk-increasing nor risk-decreasing, one may use proposition 3 and condition (6) to sign the marginal risk premium for risk-averse decision makers with convex or linear marginal utility. The results indicate that, except at low levels of input use in corn, increasing nitrogen use leads to less preferred output distributions ordered by third degree stochastic dominance. Hence, if the optimal input level falls in these ranges, the marginal risk premium is positive for producers with convex marginal utility.

The results of all these situations are summarized in tables 1 and 2.

Concluding Remarks

In the literature on output variability and input choice, two kinds of questions are frequently posed: (*a*) what is the impact of input application on output variability? and (*b*) how are risk-averse input choices different from risk-neutral decisions? The two questions are related: those analyzing the first question have looked for implications by answering the second question (Antle and Goodger; Love and Buccola; Roumasset; Smith and Umali). In a survey of the literature on the impact of fertilizers on output variability, Roumasset et al. (pp. 227) are explicit in connecting the two issues. "First, does use of fertilizer increase the variability of crop yields? Second, is the increase (if any) in variability large enough relative to increases in expected yields from nitrogen use to reduce substantially the optimal fertilizer use under risk-averse preferences relative to risk neutral optimal fertilizer use?"

Surprisingly, however, the theoretical basis for the presumed link between the impact of input use on output variability and optimal input

Table 1. Sign of the Marginal Risk Premium: Cotton

Input level (lbs)	$U'' < 0$	$U'' < 0$ and $U''' > 0$	$U'' < 0$ and $U''' = 0$
0.0	+	+	+
7.5	+	+	+
15.0	+	+	+
22.5	+	+	+
30.0	?	+	+
37.5	?	+	+
45.0	?	+	+

Table 2. Sign of the Marginal Risk Premium: Corn

Input level (lbs)	$U'' < 0$	$U'' < 0$ and $U''' > 0$	$U'' < 0$ and $U''' = 0$
0.0	?	?	–
7.5	?	?	–
15.0	?	+	+
22.5	?	+	+
30.0	+	+	+
37.5	+	+	+
45.0	+	+	+

level has not been fully explored. The present paper has filled that gap by presenting the only technological characteristics relevant in determining the impact of production risk on input use.

Implications for empirical research are twofold. First, by weakening the sufficient conditions to the maximum extent possible, the qualitative implications for a risk-averse producer's optimal input use can be inferred for all estimated technologies. If the estimated technology violates (1), the researcher can be sure that, without further information on preferences, it is impossible to predict departures from risk neutral input levels. Second, (1) is a robustness condition in empirical work which, if satisfied, guarantees the qualitative nature of the estimated magnitude (marginal risk premium) against misspecification of utility function within the class of concave functions. If the technology fails to satisfy (1), (6) serves as a robustness condition for risk averters with convex marginal utility.

A promising direction for future work would be to consider in detail the interactions between multiple inputs in the production process. To take a specific example, fertilizers could be risk increasing in the absence of irrigation but risk decreasing in the presence of irrigation. Discovering such aspects of the technology and the conditions under which they occur is important for deeper understanding of technology-driven choices.

[Received January 1991; final revision received February 1992.]

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Appendix

Proof of Proposition 1

Sufficiency: The marginal risk premium of input i is

$$RP_{x_i} = \int_{q0}^{qm} \pi(\mathbf{x}^*)F_{qx_i}(q, \mathbf{x}^*)dq - w_i.$$

Integrating by parts and recognizing that $F_{x_i}(q_m, \mathbf{x}) = F_{x_i}(q_o, \mathbf{x}) = 0$,

$$RP_{x_i} = -\int_{q0}^{qm} F_{x_i}(q, \mathbf{x}^*)dq - w_i \tag{A1}$$

where $\mathbf{x}^*$ is the solution to the following problem

$$\max_{x_i} \int_{q0}^{qm} U(q - \mathbf{w}'\mathbf{x})F_q(q, \mathbf{x})dq.$$

The first-order condition for an interior solution is

$$\int_{q0}^{qm} U(\pi)F_{qx_i}(q, \mathbf{x}^*)dq - w_i \int_{q0}^{qm} U'(\pi)F_q(q, \mathbf{x}^*)dq = 0 \text{ or}$$

$$\left[\int_{q0}^{qm} U(\pi)F_{qx_i}(q, \mathbf{x}^*)dq\right] \Big/ \left[\int_{q0}^{qm} U'(\pi)F_q(q, \mathbf{x}^*)dq\right] = w_i.$$

The denominator is expected marginal utility. Integrating the numerator by parts, the first-order condition becomes

$$-\left[\int_{q0}^{qm} U'(\pi)F_{x_i}(q, \mathbf{x}^*)dq\right] \Big/ \mathrm{E}U'(\pi) = w_i.$$

Multiplying and dividing the numerator by F_q

$$-\mathrm{E}[U'(\pi)(F_{x_i}/F_q)]/\mathrm{E}U'(\pi) = w_i$$

or

$$-\mathrm{E}(F_{x_i}/F_q) - \mathrm{cov}[U'(\pi), F_{x_i}/F_q]/\mathrm{E}U(\pi) = w_i$$

But $\mathrm{E}(F_{x_i}/F_q) = \int_{q0}^{qm} F_{x_i}(q, \mathbf{x}^*)dq$. Substituting and using (A1),

$$\text{(A2)} \qquad RP_{x_i}(\mathbf{x}^*) = \mathrm{cov}[U'(\pi), F_{x_i}/F_q]/\mathrm{E}U'(\pi)$$

Because the denominator is positive, the marginal risk premium has the same sign as the numerator, which is equal to

$$\int_{q0}^{qm} \{F_{x_i}(q, \mathbf{x}^*)/F_q(q, \mathbf{x}^*) - \mathrm{E}[F_{x_i}(q, \mathbf{x}^*)/F_q(q,\mathbf{x}^*)]\}U'(\pi)F_q(q, \mathbf{x}^*)dq.$$
$$= \int_{q0}^{qm} t(q, \mathbf{x}^*)U'(\pi)F_q(q, \mathbf{x}^*)dq$$

$$= U'(\pi)\left[\int_{q0}^{q} t(y, \mathbf{x}^*)F_q(y, \mathbf{x}^*)dy\right]_{q0}^{qm} - \int_{q0}^{qm} U''(\pi)\left[\int_{q0}^{q} t(y, \mathbf{x}^*)F_q(y, \mathbf{x}^*)dy\right]dq$$

$$= [U'(\pi)T(q, \mathbf{x}^*)]_{q0}^{qm} - \int_{q0}^{qm} U''(\pi)T(q, \mathbf{x}^*)dq$$

and because $T(q_0, \mathbf{x}) = T(q_m, \mathbf{x}) = 0$, we get

$$\text{(A3)} \qquad RP_{x_i}(\mathbf{x}^*) = -\left[\int_{q0}^{qm} U''(\pi)T(q, \mathbf{x}^*)dq\right] \Big/ \mathrm{E}U'(\pi)$$

which is positive for all risk averters if $T(q, \mathbf{x}^*) \geq 0$ for all q and negative for all risk averters if $T(q, \mathbf{x}^*) \leq 0$ for all q.

Necessity: Suppose the marginal risk premium of input i is positive for all concave utility functions. Clearly, T cannot be negative for all q. If T is positive for all q, there is nothing more to prove. So suppose $T(q, \mathbf{x}^*)$ assumes positive and negative values over the domain of q. Then there exists an interval (q_i, q_j) such that

$$\text{(A4)} \qquad \int_{qi}^{qj} T(q, \mathbf{x}^*)dq < 0.$$

But $T(q, \mathbf{x}^*)$ also satisfies

$$\text{(A5)} \qquad -\int_{q0}^{qm} U''(\pi)T(q, \mathbf{x}^*)dq > 0$$

for all concave U. For a quadratic utility function, this reduces to

$$\text{(A6)} \qquad \int_{q0}^{qm} T(q, \mathbf{x}^*)dq > 0$$

Let V be a utility function such that

(A7)

$$V(\pi) = \begin{cases} V_1 = a_1 + b_1\pi(q) - (c_1/2)\pi^2(q) \text{ for } q \leq q_i \\ V_2 = a_2 + b_2\pi(q) - (c_2/2)\pi^2(q) \text{ for } q_i \leq q \leq q_j \\ V_3 = a_3 + b_3\pi(\mathrm{q}) - (c_3/2)\pi^2(q) \text{ for } q \geq q_j \end{cases}$$

From (A3), the marginal risk premium for V is of the same sign as

$$-\int_{q0}^{qm} V''(\pi)T(q, \mathbf{x}^*)dq = c_1\left[\int_{qo}^{qi} T(q, \mathbf{x}^*)dq\right] + c_2\left[\int_{qi}^{qj} T(q, \mathbf{x}^*)dq\right] + c_3\left[\int_{qj}^{qm} T(q, \mathbf{x}^*)dq\right].$$

Choosing $c_1 = c_3 = c$, the above becomes

$$\text{(A8)} \qquad -\int_{q0}^{qm} V''(\pi)T(q, \mathbf{x}^*)dq = c\left[\int_{q0}^{qi} T(q, \mathbf{x}^*)dq + \int_{qj}^{qm} T(q, \mathbf{x}^*)dq\right] + c_2\left[\int_{qi}^{qj} T(q, \mathbf{x}^*)dq\right]$$

where the integral in the second square brackets on the RHS is negative [from (A4)] and the quantity in in the first square brackets on the RHS is positive [because of (A4) and (A6)]. So if we choose a small and positive c and a large enough and positive c_2, the marginal risk premium will be negative. But if V is concave (to be shown below), this is a contradiction. Therefore, $T(q, \mathbf{x}^*)$ cannot be negative over any subinterval of (q_0, q_m).

It remains to be seen that V is concave. This can be achieved by a suitable choice of parameters. Suppose that c and c_2 have been chosen to make (A8) negative. With a possibly discontinuous second derivative (as c_2 may not be equal to c), V is concave if it is differentiable such that V' is decreasing (Binmore, pp. 117).

From (A7), it is clear that q_i, and q_j are the only two points where the utility function could be discontinuous and nondifferentiable. Thus, the V function is continuous everywhere if $V_1[\pi(q_i)] = V_2[\pi(q_i)]$ and $V_2[\pi(q_j)] = V_3[\pi(q_j)]$. This is assured by choosing $a_1 = 0$, $a_2 = (b_1 - b_2)\pi(q_i) + (c_2 - c)\pi^2(q_i)/2$ and $a_3 = a_2 + (b_2 - b_3)\pi(q_j) + (c - c_2)\pi^2(q_j)/2$. As regards differentiability, the derivative of V within each of the subintervals is given by

$$V'(\pi) = \begin{cases} V_1' = b_1 - c\pi(q) \text{ for all } q \in (q_0, q_i] \\ V_2' = b_2 - c_2\pi(q) \text{ for all } q \in (q_i, q_j) \\ V_3' = b_3 - c\pi(q) \text{ for all } q \in [q_j, q_m). \end{cases}$$

Here, V is differentiable at q_i and q_j (and hence differentiable everywhere) if $V'(\pi)$ is continuous at q_i and q_j; i.e., $V_1'(\pi(q_i)) = V_2'(\pi(q_i))$ and $V_2'(\pi(q_j)) = V_3'(\pi(q_j))$.

So, for the c and c_2 which make (A8) negative, choose $b_3 > c\pi(q_m)$ and b_1, b_2 to satisfy

$$\text{(A9)} \qquad b_2 - c_2\pi(q_j) = b_3 - c\pi(q_j) \Rightarrow b_2 = b_3 + (c_2 - c)\pi(q_j)$$

and

$$\text{(A10)} \qquad b_1 - c\pi(q_i) = b_2 - c_2\pi(q_i) \Rightarrow b_1 = b_2 + (c - c_2)\pi(q_i).$$

Notice that continuity and the fact that V' is decreasing in each subinterval mean that V' is decreasing on the entire interval (q_0, q_m). Further, because b_3 is picked to be greater than $c\pi(q_m)$, V' at q_m is positive. But that is the minimum value of V', and so V' is positive for all q.

PROOF OF PROPOSITION 2: Suppose $t(q, \mathbf{x_0})$ has one root (say, q_1) over the domain (q_0, q_m) such that $t > 0$ for $q \in (q_0, q_1)$ and $t < 0$ for $q \in (q_1, q_m)$. We need to show $\int_{q0}^{q} t(y, \mathbf{x_0})F_q \geq 0$ for all q. Clearly $\int_{q0}^{q_1} t(y, \mathbf{x_0})F_q \geq 0$. But $\int_{q0}^{q_m} t(y, \mathbf{x_0})F_q = 0$. So if for any $q > q_1$, $\int_{q0}^{q} t(y, \mathbf{x_0})F_q \leq 0$, then t must be positive beyond q_1 in order for the integral to rise to zero at q_m. But that contradicts our supposition that t is negative for $q > q_1$. Hence $\int_{q0}^{q} t(y, \mathbf{x_0})F_q \geq 0$ for all q. The proof is similar for the other case where $t(q, \mathbf{x})$ changes sign from negative to positive.

PROOF OF PROPOSITION 3: As shown by (A3), the marginal risk premium is of the same sign as $-\int_{q0}^{q_m} U''(\pi)T(q, \mathbf{x}^*)dq$. Integrating by parts, this quantity is

$$-U''(\pi(q_m))\int_{q0}^{q_m} T(q, \mathbf{x}^*)dq + \int_{q0}^{q_m} U'''(\pi)\left[\int_{q0}^{q} T(y, \mathbf{x}^*)dy\right]dq$$

which is positive if $\int_{q0}^{q} T(y, \mathbf{x}^*)dy \geq 0$ for all q and negative if $\int_{q0}^{q} T(y, \mathbf{x}^*)dy \leq 0$ for all q.